Examination Of Gifted Students' Probability Problem Solving Process In Terms Of Mathematical Thinking

[1] Assist. Prof. Dr., Ahi Evran University Faculty of Education, Elementary Mathematics Education Department,
E mail: serdalbaltaci@gmail.com

Serdal BALTACI [1]

ABSTRACT

It is a widely known fact that gifted students have different skills compared to their peers. However, to what extent gifted students use mathematical thinking skills during probability problem solving process emerges as a significant question. Thence, the main aim of the present study is to examine 8th grade gifted students' probability problem-solving process related to daily life in terms of mathematical thinking skills. In this regard, a case study was used in the study. The participants of the study were six students at 8th grade (four girls and two boys) from the Science and Art Center. One of the purposeful sampling methods, maximum variation sampling was used for selecting the participants. Clinical interview and problems were used as a data collection tool. As a results of the study, it was determined that gifted students use reasoning and strategies skill, which is one of the mathematical thinking skills, mostly on the process of probability problem solving, and communication skills at least.

Keywords: *Gifted students, Mathematical thinking skills, Probability.*

INTRODUCTION

When we encounter uncertainty in positive sciences and in our daily lives, we are subject to using probability consciously or unconsciously, if required. This makes probability even more significant for individuals working in various professions (Hirsch & O'Donnell, 2001). Notwithstanding, probability which is a foremost concept is ascertained as one of the most problematic topics in terms of both students and teachers (Batanero & Serrano, 1999; Batanero, Serrano & Garfield, 1996; Dooren, Bock, Depaepe, Janssens & Veschaffel, 2003; Kafoussi, 2004; Munisamy & Doraisamy, 1998; Yildiz & Baltaci, 2015). The NCTM (2000) emphasized that students as conscious citizens and smart-consumers need to know probability topics with the aim of reasoning.

When examining the reasons for probability topics being hard to understand, Borovcnik and Kapadia (2009) identified that students answer the questions with intuition rather than logic while making sense of the probability concepts and thus face difficulty. Batanero, Chernoff, Engel, Lee, and Sánchez (2016) suggested that instead of understanding the topics, a large majority of the students try to memorize formulae and are unable to understand the questions. In addition, they have a negative attitude toward these concepts and do not use proper teaching materials. Carpenter, Corbitt, Kepner, Lindquist, and Reds (1981) stated that the lack of students' prior knowledge and skills prevents effective learning of probability concepts. O'Connell (1999) also expressed that students make mistakes because of the conceptualism while solving probability problems. In order to resolve this sort of problems, several studies in the relevant literature emphasize adopting an experiment-based mindset of teaching, studying on stochastic events to help students apprehend the theoretical aspect of probability (Watson, 2006) and using student-centered approaches

(Aspinwall & Shaw, 2000; Gürbüz, 2008; Polaki, 2002; Tatsis, Kafoussi & Skoumpourdi, 2008). In addition, several institutions have stated that it is necessary to raise awareness about probability practices and it is important to use technology for developing conceptual understanding (Guidelines for assessment and instruction in statistics education, 2005; National Council of Teachers of Mathematics, 2000). According to Mills (2002) and Gürbüz (2008), difficulties in learning probability can be overcome using technology. Whether gifted students have these negativities or not, and if any what are the reasons are required to be answered in detail.

The concept of giftedness is defined by various educators and different parameters are explained. Experts define gifted children or students as those having high performance related to intelligence, creativity, art, or leadership capacity or specific academic areas in comparison to their peers (Kirk & Gallagher, 1989). The gifted are individuals differing in terms of the distribution of human characteristics, frequency, timing and composition (Akarsu, 2001; Meyen & Skrtic, 1988). Therefore, that gifted students' qualitative and quantitative thinking skills are developed makes it possible for them to have better problem solving skills than the typical students (Knepper, Obrzut & Copeland, 1983) as problem solving does not solely mean to solve the easy tasks faced by individuals in daily life. Meanwhile, the highest cognitive functions such as analyzing, generalizing and synthesizing are used with regard to problem solving method (D' Zurilla, Nezu & Maydeu-Olivers, 2004; Henson, 1993; Naglieri & Dass, 2005). Hence, solving problems fast as well as memorizing symbols, numbers and formulae are not to be considered as an indicator of mathematical giftedness (Wieczerkowski, Cropley & Prado, 2000). Mathematical thinking skills can be a criterion for mathematical giftedness.

As in Turkey, the development of students' mathematical thinking skills among the overall objectives of mathematics teaching is emphasized in all countries (Keith, 2000; Mason, Burton & Stacey, 1998). Mathematical thinking can be summarized as a method of accessing from the known to the unknown which consists of making assumptions, gathering evidence and generalization processes regarding the cases (Baki, 2008). In order to participate actively in the mathematical thinking process, what is needed is to have mathematical thinking skills or develop them. Suzuki (1998) gathered these skills under five titles including conceptual knowledge, procedural knowledge, reasoning and strategies, maturity and communication.

Literature review shows that Suzuki's mathematical thinking skills classification is more comprehensive and broad compared to that of other researchers. For instance; reasoning and strategies defined by Suzuki comprise guessing, association and persuasion – proof processes which are described by the other researchers. Thus, the skills identified by Suzuki are taken into account. These skills are described briefly as follows.

Conceptual Knowledge: Not only understanding the definition of the concepts but also mutual transitions and relations between the concepts are expressed (Baki, 2008). In other words, it is a skill including such features as understanding the problem statement, various meanings and the interpretations of the concepts as well as recognizing the problem with the unknown and equivalent quantities (Suzuki, 1998).

Procedural Knowledge: It includes strategies and methods necessary for implementing the concepts and principles (Taconis, Ferguson-Hessler & Broekkamp, 2001). In addition, Suzuki (1998) revealed that procedural skill includes such operations as making numerical operations and algorithms accurately, implementing the solution plans and controlling each action.

Reasoning and Strategies: This skill covers various behaviors such as demonstrating reasoning ability, selecting appropriate strategies, evaluating and interpreting the results and operations (Suzuki, 1998). On the other hand, Umay (2007) explained this ability as a process by which a rational decision is ensured taking into consideration all probabilities through evaluating the process with the knowledge available.

Maturity: Maturity is described as a skill including various behaviors such as organizing comprehensive solution strategies, changing the locution of the problem, and so forth in terms of knowledge development (Suzuki, 1998).

Communication: It is a skill which covers such behaviors as using mathematical language so that ideas can be transmitted as required, explaining the mathematical logic and reasoning for the solution process, behavior as well as showing the connections between mathematical ideas (Suzuki, 1998).

The studies on the problem solving processes of gifted students generally focus on how gifted students at secondary schools solve the mathematical non-routine problems (Garofalo, 1993; Sriraman, 2003). Furthermore, most of the studies available in the literature said are quantitative studies (Suzuki, 1998; Tasdemir, 2008; Tuna, 2011). It has been found that how students use mathematical thinking skills while solving probability problems was not examined, although it is a widely known fact that gifted students have different skills compared to their peers. However, to what extent gifted students use mathematical thinking skills during probability problem solving process emerges as a significant question. Therefore, the main aim of the present study is to examine 8th grade gifted students' probability problem-solving process related to daily life in terms of mathematical thinking skills. In this context, the research question is: "*how do gifted students use mathematical thinking skills while solving probability problems related to daily life?*"

METHODOLOGY

In this section; research method, participants of the study, application process, data collection and analysis are presented.

Research Method

A case study method was used in order to provide opportunity for examining a particular group in-depth and to examine data obtained from the data collection tools without any concern for generalization.

Participants

Gifted students are trained at the Science and Art Center which is a different educational institution independently from the school program (SAC) in Turkey. The selection process of gifted students who will be trained at these centers takes place in 6 stages which are classified as diagnosis, nomination, pre-assessment, group scanning, individual examination, registration and placement. These stages may indicate that Turkey attaches importance to the selection of gifted students.

In the current study, one of the purposeful sampling methods, maximum variation sampling was used in order to identify the participants. The aim of the maximum variation sampling is to create a small sample relatively and to reflect the individuals' diversity to a maximum degree who will favor the research problem in this sample (Creswell, 2005; Johnson & Christensen, 2004). In this sense, the participants consist of six students at 8th grade continuing their education at Science and Art Center. Four of them are starting to train at Science and Arts at Center 6th grade have been getting education at this center for 3 years while two participants started there at 5th grade and have been taking education for 4 years. Four of the gifted students participating in the research are female students while two are male. It was paid utmost attention to selecting those who have higher problem-solving skills, are able to use GeoGebra software and volunteer for the interview, in accordance with teachers' opinions.

Dynamic softwares provides a significant place in learning processes of students (Tatar, 2012). GeoGebra, which is one of the dynamic software, transfers mathematical symbols, graphics and obtained values to table via various Windows (Aktümen, Horzum, Yıldız & Ceylan, 2010). At the same time, GeoGebra dynamic mathematics software helps students display their high-level thinking skills (Edwards & Jones, 2006).

Data Collection Tools

Clinical interview and problems were used during the data collection process. Before the study was conducted, questions which can be asked during clinical interviews were determined. During the determination of the problems available in the data collection tool, their appropriateness to the level of students along with the demonstration of mathematical thinking skills in each problem was noted. The problems included dependent and independent probability and geometric probability topics. Accordingly, the problems included in the study were prepared by means of mathematics curriculum, math textbooks and books for teaching mathematics, then it was noted that the problems are approximately the same level with those taught at the Science and Art Center by discussing them with the mathematics teachers. Then, clinical interview questions and problems were independently evaluated by two experts. After this evaluation process, evaluation reports were aggregated and discussed, and the final form of the interview questions and problems was created.

Implementation Process

Before starting the implementation, we conferred with three teachers working at the Science and Art Center and received information about how they teach probability there. They opined that gifted students learn the concept of probability principally at school. At the Science and Art Center, the teachers ask more strategy-requiring questions and provide students with a computer-aided environment to improve their reasoning. The teachers stated that they use GeoGebra in teaching probability concepts.

A pilot study was conducted with two gifted students with the purpose of identifying the flaws that may occur in the study. After the required arrangements following the pilot study, the original implementation was started. Thus, the problems included in the study were formed in the Appendix. During the implementation, asking the students probability problems respectively, their mathematical thinking skills were studied in-depth through clinical interview. During this process, environment was designed in such a way that students can use both paper – pen and GeoGebra software.

Before starting the clinical interview, the students and the researcher spent time together and the researcher informed them about the purpose of the research superficially. Each interview was recorded via a digital voice recorder with the permission of the students. The interviews were conducted individually and each lasted approximately 70 minutes. The interviews were carried out at guidance service which is a quiet area where the students feel unworried. Besides, during the interviews, the processes students did and the models were recorded on both GeoGebra screen and paper-pen environment associated with the students' problems.

Data Analysis

Strauss and Corbin (1990) refer to three types through which qualitative data may be coded. These types of coding are; coding based on the predetermined concepts, coding depending on the concepts derived from the data, coding based on a general framework (Punch, 2013). In the study which takes into consideration Suzuki's (1998) classification related to mathematical thinking skills and the definitions, coding based on the predetermined concepts was done. This coding emphasized findings section as conceptual knowledge (CK), procedural knowledge (PK), reasoning and strategies (RS), maturity (M), and communication (C). On the other hand, while forming frequency tables presented at the end of each problem, Suzuki (1998) mathematical thinking skills were paid attention and the sum total was noted.

Prior to data analysis, data base dump and control were enacted. Each interview was noted by paying attention to the interviewer-interviewee written order, phonetically and without any correction. Next, the themes were separately developed by the researcher and a field expert. Therefore, the related information was given to a field expert, too. To shape the analysis part into final form, the independent series of analysis was brought together and discussed. This was a necessary process to provide the validity of the study. Thus, how gifted students demonstrate each skill during probability problem solving process was identified in detail. Intercoder reliability was 95%. Moreover, various codes were used while the findings were presented. In this context; A refers to a researcher and G1 signifies gifted students 1 ...and so forth.

FINDINGS

In this section, 8th grade gifted students' mathematical thinking skills used for solving probability problems related to their daily lives were analyzed. The results were supported via direct quotes from the clinical interview, the solutions students found and the models on GeoGebra screen. Thus, the findings were identified as follows:

The findings related to the first problem solving process:

It was found that all of the gifted students stated the problem with their own words, and then they tried to express their ideas about solving the problem by putting forward the given and required. For instance, G1 stated that a circle tangent to the two opposite sides is drawn inside a rectangle the perimeter of which is 24 units and expressed that she wants to find the probability of hitting the arrow outside the circle by determining the given and the required.

G1: The problem is about taking a quadrilateral and a circle tangent to the two sides of this quadrilateral. However, we are expected to hit the arrow outside the circle.

A: Yes. What can we do?

G1: Now, we should have a quadrilateral whose perimeter is 24 units. We will draw a circle inside this quadrilateral which is tangent to the two opposite sides. Then, we will calculate the probability of hitting outside the circle. I think the given and the required related to the problem are in this way.

A: OK. Let's continue.

G1: And here quadrilateral must be special. It can be a square.

As observed above, G1 tried to explain the problem with her own words by determining the given and the required and decided to start from square a particular case in order to solve the problem (CK). This shows the understanding of the problem statement which is an indicator of the conceptual knowledge. Afterwards, all of the gifted students who think that the target is a square organized the instructions presented for the problem solving; besides, they modeled the required in the paper and pencil environment. Thinking that the model that he created in this process is correct, G3 has tried to solve the problem as follows.

G3: If the target was a square, the diameter of the circle that we will draw inside it would be 6 units. If the selected point was inside, the required probability would be 9π / 36; we get the result when we subtract it from 1.

A: So is this our result?

G3: Here our global cluster, all the results, is the area of the square and the outside of the circle is the required. Actually, it is like throwing money experiment. So all the results are tail or head. If head is required, then we divide one situation into two, which is also observed here. But there is only one situation that is Geometric. In fact, the probability of 3 out of 4 hits related to the probability of being inside is ¾ while the probability of being outside 1 out of 4 hits is ¼. The result is shaded area / the entire area = ¼. When we get 3 instead of π, as the area of the circle will be 36, the probability of being inside would be 27/36. We can find the result when we subtract it from 1.

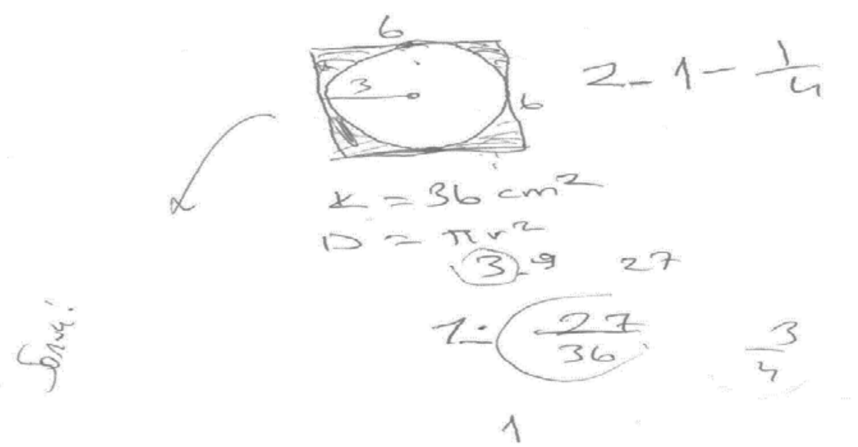

FIG. 1. A section related to the solution of G3 student in the event of a square target

As is observed from the figures and interview data, G3 tried to do the processes with self-confidence which means that he achieved the solution fast through practicing (PK; M). Moreover, G3 conveyed the ideas as required through giving an example related to the solving process, thus having an effort to use mathematical language (C). Hence, he rationalized the problem by modifying the expression of the problem (M). Trying to check the accuracy of what she did, G2 stated that she can control the result by solving it again without GeoGebra software; however, the other five gifted students wanted to check the result via GeoGebra software. An example representing the processes conducted by G5 on software is as follows.

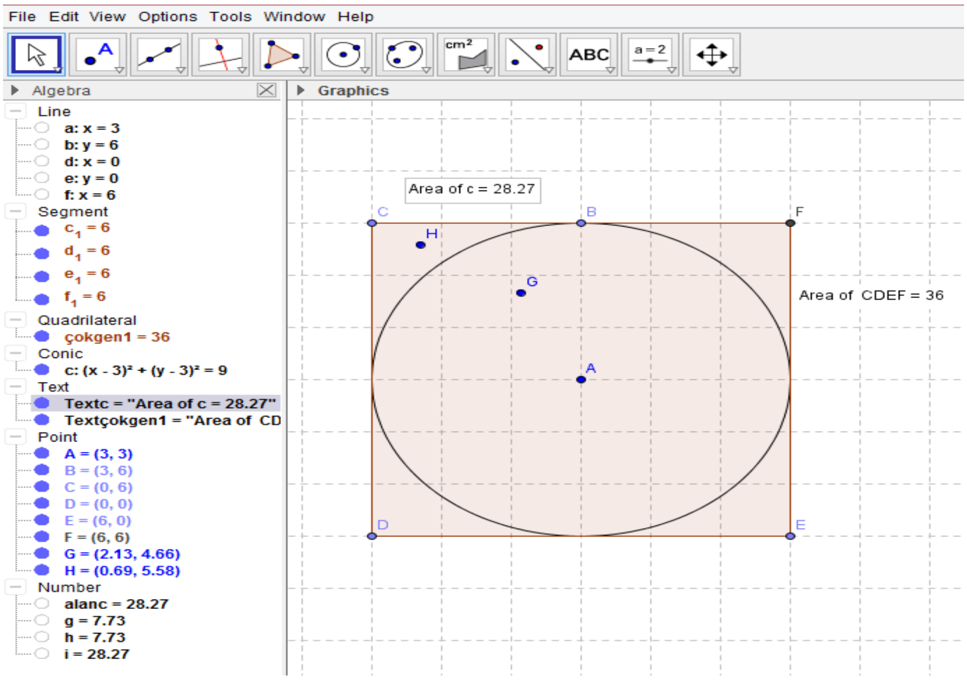

FIG. 2. G5's checking process of the result on software in case of a square target

As seen in Fig. 2, G5 tried to check the result he found via software (RS). During this process, G5 expressed the roundings done in the paper and pen environment on the screen and made an effort to do the processes by means of software using mathematical language (PC). As a result, following the process presented above as a section, all the gifted students found the correct result in case of a square target. Thereafter, when it comes to a rectangle target, it was clarified that the majority of the gifted students reached the solution through generalizing the required probability mathematically (CK). Ultimately, all the gifted students managed to find the result in the event of a rectangle target. The figure drawn by G4 to express the required probability in general is as follows:

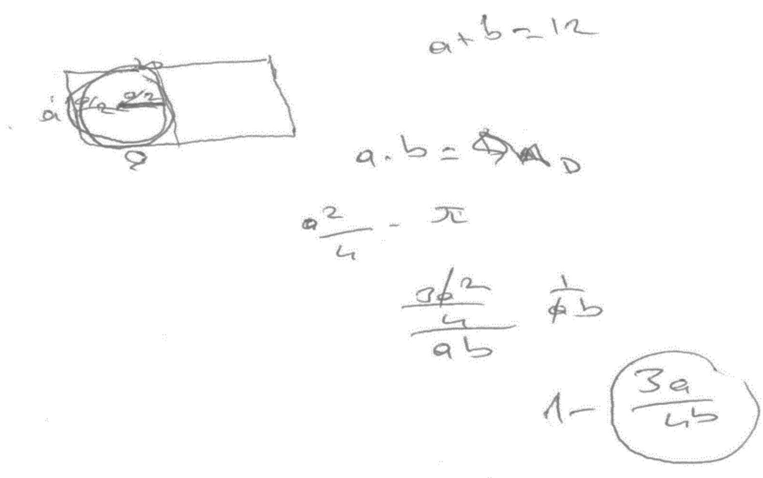

FIG. 3. A section related to G4's generalization the required probability mathematically in case of a rectangular target

Subsequently, G4 determined the edges of a rectangle as 4 and 8 units and tried to explain the result as follows (PK). Given the process followed above, it was revealed that G4 did the processes in a meaningful way in case of a rectangle target (CK), announced these expressions through using a mathematical language (PK; C), reflected her reasoning ability for choosing the appropriate strategy and got the result in a systematic way (RS; M). Having tried to present the accuracy of the result in case of a target square, gifted students stated that they are sure of what they did in case of a rectangle target (RS; C).

In the event of a square and rectangle target, some of the successful gifted students did not achieve the same success in the paper and pen environment as in the case of a parallelogram target. In fact, it is hard to find the required probability for the parallelogram target. However, four of the gifted students were successful in solving this problem. As the students who are successful, G2 also found the height of the parallelogram with a special value in the paper and pen environment and found the correct result (CK; PK). This process is as follows.

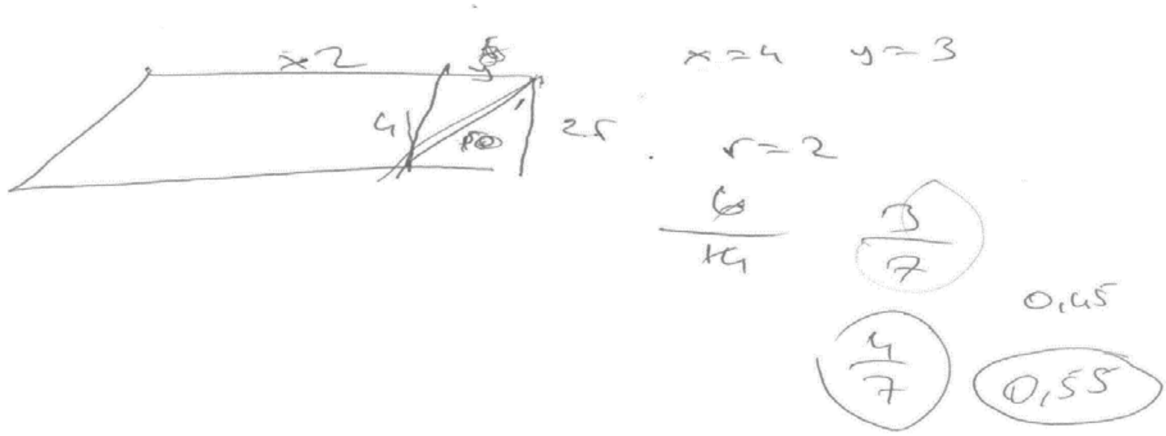

FIG. 4. G2's achieving the result with giving special values to the edges in case of a parallelogram target

As considered above, the gifted students got a specific triangle with the aim of finding the height of the parallelogram gives us clues regarding their way of thinking. On the other hand, unlike the other two students who solved the problem successfully, G6 generally expressed the edges of parallelogram and generalized the required probability mathematically based upon these data (CK; PK; M). Thereafter, G6 forming the required on GeoGebra screen found the height through taking different numbers for the edges of parallelogram and noted her findings in the paper and pen environment (M; C). The dialogs and figures related to this process are as follows.

G6: If we get the edges of it as a and b, the probability of being inside is $\pi r/(2b)$ when the radius of the circle is r, the area of it is πr^2 and the area of parallelogram is 2r.a. I find the required result while subtracting it from 1. Yet, I cannot find the height when I consider the edges of parallelogram as 4 and 8 units, but I can achieve the required through GeoGebra.

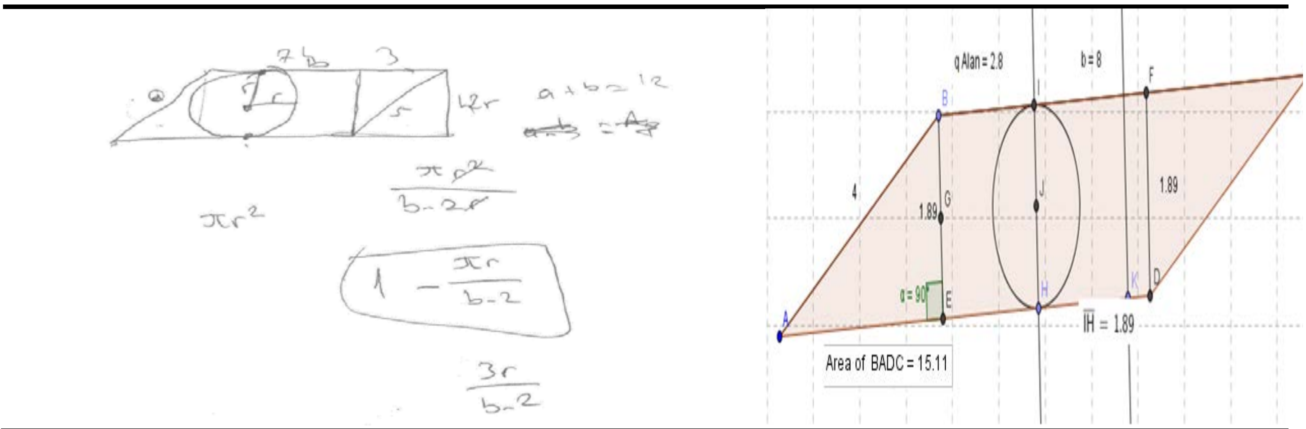

FIG. 5. A section of G6’s generalization and modeling on software the required probability in case of a parallelogram.

Within the scope of the present study the aim is to examine gifted students' mathematical thinking skills during the probability problem solving process, the frequency table presented as Table 1 shows how many times each student uses mathematical thinking skills during every interview process regarding all the problems.

Table 1. Frequency Table Related to The First Problem

	G1	G2	G3	G4	G5	G6
Conceptual Knowledge (CK)	18	17	17	17	18	18
Procedural Knowledge (PK)	16	15	16	17	15	14
Reasoning and Strategies (RS)	19	18	18	19	19	20
Maturity (M)	14	15	16	17	16	17
Communication (C)	13	14	14	15	13	14

When Table 1 is examined, it was observed that the reasoning and strategies are mostly used in this process, which is followed by conceptual knowledge communication skills. The first problem is seen as much more difficult perhaps because: geometric probability and the background knowledge required for the problem is quite substantial.

The findings related to the second problem solving process:

All the gifted students were able to easily understand that the problem is a probability problem. Afterwards, students entered in the process of problem solving, making comments about which type of probability is asked related to the problem (CK; M). In this

process, the dialog between the researcher and G1 is as follows.

G1: The probability that Ali hit is 40%, 2/5 while that of Veli's hitting is 75%, ¾. The probability of being hit by both of them and by one of them is required. It refers to an independent situation as their hitting the target does not affect each other. Thus, the answer of the option a is 6/20.

A: Why? Where did you get this answer?

G1: What is required is the hitting of both. That is why, I multiply their hitting probabilities. For instance, two persons throw the coins and both of them are heads.

As is seen, explaining that the problem is about the probability topic, G1 expressed the solution correctly (CK; M). In fact, G1 tried to explain the solution in another way through mathematical reasoning (C). In addition, as shown in the example a section of which is presented, the fact that all of the gifted students refer the problem as the probability problem gives us various clues concerning their success in problem-solving strategies.

Thereon, of all the gifted students trying to figure out the probability of being hit by only one, G3 was unable to state the required probability at first (CK) whereas the others did not have difficulty in finding the probability. In this process, G3 tried to find the result by adding up hitting probability of them and vice versa, then got the result realizing his mistake. This process is as following:

$$\frac{2}{5}\cdot\frac{3}{4}=\frac{7}{10} \quad \checkmark \quad \frac{3}{5}\cdot\frac{1}{4}=\frac{3}{20}$$

$$\frac{3}{20}+\frac{3}{10}=\frac{9}{20}$$

$$\overset{+}{\frac{2}{5}}\cdot\overset{-}{\frac{1}{4}}+\overset{-}{\frac{3}{5}}\cdot\overset{+}{\frac{3}{4}}=\frac{2}{20}+\frac{9}{20}$$

$$=\frac{11}{20}$$

FIG. 6. A section of the probability problem where G3 did wrong in calculating the required probability

G3: I did wrong here. One will hit while the other will not so the result is 2/5.1/4 + 3/5.3/4 = 11/20.

G3 found the false result without understanding the problem through adding up hitting probability of both and vice versa (CK; PK). Following, being aware of the mistake he made, G3 was able to find hitting probability of one (RS).

Table 2. Frequency Table Related to The Second Problem

	G1	G2	G3	G4	G5	G6
Conceptual Knowledge (CK)	13	12	11	12	13	10
Procedural Knowledge (PK)	11	10	9	11	10	11
Reasoning and Strategies (RS)	10	11	10	12	11	10
Maturity (M)	9	8	6	7	8	7
Communication (C)	8	7	7	6	7	8

It was revealed that conceptual knowledge is mostly used in this process, which is followed by reasoning and strategies and communication skills. The greater use of conceptual knowledge may be because they expressed the given problems related to daily life as probability problems and carry out the processes in a meaningful way.

The findings related to the third problem solving process:

As for the other problem, the students are inclined to solve the problem as immediately as possible. To illustrate, G5 stated he encountered such a problem before and was able to find the solution as follows. The dialog and the solution found by G5 are as follows:

G5: Actually, I encountered with this kind of problem before. Here we'll draw a white ball from bag A and put it into bag B. When we draw a ball from bag B, we want it to be a white ball.

A: OK. How were the problems you have encountered before?

G5: It was almost the same; that is there was a bag and we draw a ball.

A: Well, how will you do it?

G5: The probability of drawing a white ball from bag A is 3/7. If we put it into bag B, then the probability of drawing white will be 3/6-1/2- as 6 balls are available. Similarly, if we draw black from bag A, the probability will be 4/7. When we put it into bag B, the probability of drawing 2 white balls out of 6 will be 2/6 -1/3-. Hence, the required probability will be 3/14 + 4/21.

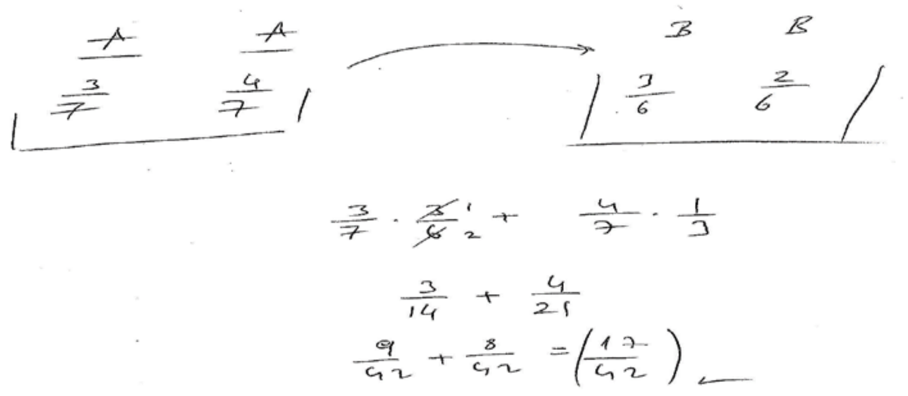

FIG. 7. A section related to the G5's solving process of the required probability

When the dialog between G5 and the researcher along with the student's solution are examined, G5 was found to easily understand the problem and started to solve it by detailing in a systematic way (CK; M). Likewise, G5 was observed to conduct the processes in an efficient manner as well as expressing ideas as necessary through using mathematical language (PK; C). The fact that gifted students expressed their ideas using mathematical language reveals the differences in their solutions.

In the process, G4 drew a figure like a probability tree, which is an example of a different strategy (M). The solving process of G4 is as the following.

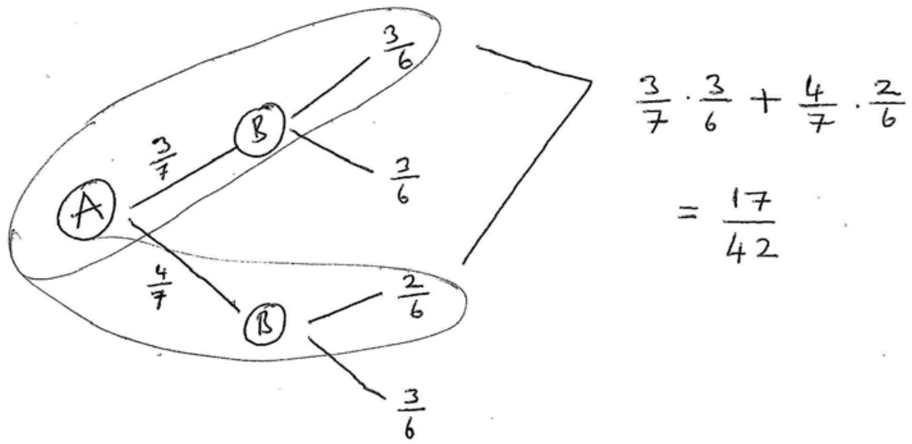

FIG. 8. A section related to G1's different solving strategy of the required probability.

As seen in Fig. 8, G4 tried to use a different problem-solving strategy (M). It may be natural for gifted students to have different thinking styles.

The majority of gifted students who are sure about the solutions they found were not involved in the process of controlling the obtained results (RS). For instance, the dialog between G6 and the researcher after finding the correct result is as follows.

A: Did you think that the result I have found is accurate? How can you prove?

G6: I think it is correct. I'm sure what I did.

Table 3. Frequency Table Related to The Third Problem

	G1	G2	G3	G4	G5	G6
Conceptual Knowledge (CK)	8	7	7	6	7	8
Procedural Knowledge (PK)	7	8	8	7	10	9
Reasoning and Strategies (RS)	8	8	7	6	8	8
Maturity (M)	6	6	7	6	8	8
Communication (C)	6	5	5	5	3	4

As for the third problem, it was revealed that operational knowledge is mostly used in this process, which is followed by reasoning and strategies and communication skills. Operational skill is at the forefront in terms of the third problem since it is likely that gifted students stated their encounter with these types of problems and did the processes efficiently. The less use of communication skills shows that gifted students focused on the solving process the minute that the problem is presented. Taking into account all these mathematical thinking skills, a frequency table is created (Fig. 9).

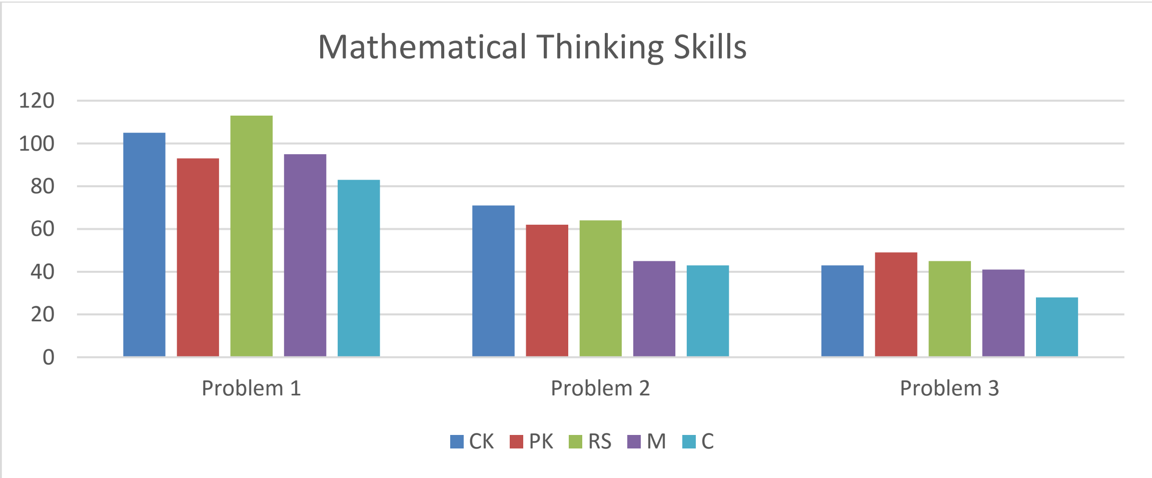

FIG. 9. The frequencies of gifted students' demonstration of mathematical thinking skills for each problem

DISCUSSION

It was found that all of the gifted students stated the problem in their own words, and then tried to express their ideas about solving the problem by putting forward the given and required. Mayer (1982) indicated that the most difficult stage of the problem for students is the understanding process. Gifted students are good at expressing the problem depending upon their own way of thinking. In parallel to the findings of the research, Garofalo (1993) and Sriraman (2003) signified that the gifted students succeeded in interpreting the problem with their own sentences, yet they wasted time during this process. This situation is an indicator of conceptual knowledge as the students' effort to understand the problems deeply. A gifted student who is willing to solve the problem correctly is supposed to understand the problem, try various ways and reach the solution.

It has been determined within the scope of the studies that the use of dynamic software by the students will benefit them when they do mathematical generalizations (Baltaci & Yildiz, 2015; Cha & Noss, 2001; Santos-Trigo & Cristóbal-Escalante, 2008). In the current study, it was pointed out that gifted students want to do the processes through qualitative detailing and thus have differences in terms of their thinking skills. On the other hand, all the gifted students were able to associate the second problem with the probability topic. Therefore, of all the skills, conceptual knowledge is the most demonstrated one by the gifted students related to the second problem. It is emphasized that students make associations in today's mathematics education programs (Chapman, 2012; NCTM, 2000). As in the present study, Putter-Smits, Takonis and Jochems (2013) and Rose (2012) determined that presenting the problems through associating them with daily life is to be effective in the students' understanding.

Considering operational knowledge skill, it was shown that gifted students did operations with full self-confidence and observed the rounding done in paper-pen environment on software screen easily. Besides, it was observed that gifted students generalized the required probability value mathematically and then reached the result by numbering the equation differently especially in terms of geometric probability problems. Students applied computational operations and algorithms easily to the mathematically generalized equations. Moreover, the gifted students tended to solve the third problem. Stating that s/he encountered with a similar problem before, one of the students initiated the solving process, which has led the gifted students to use operational knowledge mostly. Yesildere and Turnuklu (2007) also made it clear that students are much more successful regarding operational problems which might use the knowledge directly rather than those using reasoning skills.

When it comes to the reasoning skills and strategies, it was emphasized that gifted students could not only interpret but also evaluate the processes they conducted as well as the results they found as analytical and algebraic through algebra screen thanks to the graphic display of the GeoGebra software. Baki, Yıldız and Baltaci (2012) state that GeoGebra dynamic mathematics software that provides functionally different displays on the same window is a key for the emergence of this finding.

Ensuring that they solve the second and the third problems correctly, the gifted students did not have a tendency for controlling the results obtained by identifying the appropriate strategies. Indeed, Lester, Garofalo, and Kroll (1989) also observed that the gifted students who carried out a successful plan during the problem-solving process did not control the results. Further, Pajares (1996) stated that gifted students had more self-efficacy beliefs while solving the problems. However, Yildiz, Baltaci and Guven (2011) determined that gifted students were unwilling to check the accuracy of the results after solving the problems. As the gifted students have self-confidence in preparing a good plan for any problem they encountered, they are not eager for illustrating the accuracy of the results. Thus, it is essential that students are required to prepare a good plan before providing them for doing crosschecking.

As for the maturity skills, all of the gifted students organized the instructions presented for the problem solving; besides, they modeled the required in the paper and pencil environment. The fact that gifted students solve the problems by drawing figures thus achieving the solutions easily is in parallel to the finding by Hong (1993) about the expression of the problem much better through creating models. As for the first problem, it is emphasized that the majority of the gifted students succeeded in the case of square and rectangle target get a specific triangle so as to find the edges of the parallelogram and found its height. Similarly, Shore and Dover (1987) stated that the thinking processes of gifted students are different from the others. For example, that the gifted students got a specific triangle with the aim of finding the height of the parallelogram gives us clues regarding their way of thinking.

It is clear that gifted students understand the problems easily and initiated the solving process detailing it in a systematic manner. Congruent with the findings of Montague, Bos and Doucette (1991), gifted students solved the problem as soon as they encountered it. Sisk (1987) also confirmed that gifted students are to choose the appropriate strategy and make original comments based upon the problem. Still, to solve the problem fast and to memorize symbols, numbers and mathematical formulas are not being considered as an indicator of giftedness (Wieczerkowski, Cropley, & Prado, 2000).

Considering the communication skills, it was ascertained that gifted students are able to use mathematical language easily while solving the problems and transferring thoughts. Cai (2003) determined that gifted students chose appropriate solving strategies and used an open communication style which represents the solutions selected during the solving process. That gifted students use mathematical language easily while transferring their thoughts shows their skill for expressing ideas mathematically as well. Accordingly, teachers are required to prepare a well-designed learning environment so that students can develop the use of mathematical language skills. As a reference to the learning environments, teachers should evoke the students' metacognition in problem solving environments as in Yildiz (2013).

In the present study, gifted students' mathematical thinking skills during probability problem solving process were observed. As discussed before, gifted students have been found to have different behaviors related to the skills in this process. Thus, teachers who want to provide a more efficient lesson environment can determine students' mathematical thinking skills in a better way and use GeoGebra software as a tool.

REFERENCES

Akarsu, F. (2001). *Üstün yetenekli çocuklar [Gifted Students]*. Ankara, Turkey: Eduser Publishing.

Aktümen, M., Horzum, T., Yıldız, A., & Ceylan T. (2010). Bir dinamik matematik yazılımı: GeoGebra ve ilköğretim 6-8. sınıf matematik dersleri için örnek etkinlikler. Retrieved from http://w3.gazi.edu.tr/web/aktumen/dosyalar/geogebraturkiye.pdf

Aspinwall, L., & Shaw, K. L. (2000). Enriching students' mathematical intuitions with probability games and tree diagrams. *Mathematics Teaching in the Middle School*, *6*(4), 214-220.

Baki, A. (2008). *Kuramdan uygulamaya matematik eğitimi [From theory to practice mathematics education]*. Trabzon, Turkey: Harf Publishing.

Baki, A., Yildiz, A., & Baltaci, S. (2012). Mathematical thinking skills shown by gifted students while solving problems in a computer-aided environment. *Energy Education Science and Technology Part B: Social and Educational Studies, Special Issue*, pp. 993-995.

Baltaci, S. (2014). Dinamik matematik yaziliminin geometrik yer kavraminin ogretiminde kullanilmasinin baglamsal ogrenme boyutundan incelenmesi [An investigation of the use of dynamic mathematics software in teaching the concept of locus in terms of contextual learning] (Unpublished doctoral thesis), Karadeniz Technic University, Trabzon, Turkey.

Baltaci, S., & Yıldız, A. (2015). Matematik öğretmen adaylarının GeoGebra yazılımı yardımıyla analitik geometrideki bir konuyu öğrenme süreçleri [Preservice mathematics teachers processes of learning an analytic geometrical subject by the help of GeoGebra software]. *Ahi Evran University Journal of Kırsehir Education Faculty (KEFAD)*, *16*(3), 295-312.

Batanero, C., Serrano, L., & Garfield, J. B. (1996). July 8-12. Heuristics and biases in secondary school students´ reasoning about probability. *Proceedings of the Conference of the International Group for the Psychology of Mathematics Education, 20th.* Valencia, Spain.

Batanero, C., & Serrano, L. (1999). The meaning of randomness for secondary school students. *Journal for Research in Mathematics Education, 30*(5), 558-567.

Batanero, C., Chernoff, E. J., Engel, J., Lee, H. S., & Sánchez, E. (2016). Research on teaching and learning probability. In *Research on teaching and learning probability* (pp. 1-33). Springer International Publishing.

Borovcnik, M., & Kapadia, R. (2009). Research and developments in probability education. *International Electronic Journal of Mathematics, 4*(3).

Cai, J. (2003). Singaporean students mathematical thinking in problem solving and problem posing: An exploratory study. *International Journal of Mathematics Education in Science and Technology, 34*(5), 719-737.

Carpenter, T. P., Corpitt, M. K., Kepner, H. S., Lindquist, M. M., & Reys, R. E. (1981). What are the chances of your students knowing probability? *The Mathematics Teacher, 73,* 342-344.

Cha, S., & Noss, R. (2001). Investigating students' understanding of locus with dynamic geometry. *Proceedings of the British Society for Research into Learning Mathematics, 21*(3)*,* 84-89.

Chapman, O. (2012). Challenges in mathematics teacher education. *Journal of Mathematics Teacher Education, 15*(4)*,* 263-270.

Creswell, J. W. (2005). *Educational research: Planning, conducting, and evaluating qualitative and quantitative Research* (2nd ed.). Upper Saddle River, NJ: Pearson Education.

Dooren, W. V., Bock, D. D., Depaepe, F., Janssens, D., & Verschaff el, L. (2003). The illusion of linearity: Expanding the evidence towards probabilistic reasoning. *Educational Studies in Mathematic, 53,* 113-138.

D'Zurilla, T. J., Nezu, A. M., & Maydeu-Oliveras, A. (2004). Social problem solving: Theory and assesment. Ec. Chang, T. J. D'Zurilla & L. J. SaFnna (Eds.), *Social problem solving: Theory, research and training*. Washington DC: American Psychological Association*.*

Edwards, J. A., & Jones, K. (2006). Linking geometry and algebra with GeoGebra. *Mathematics Teaching, 194,* 28-30.

GAISE. (2005). *Guidelines for assessment and instruction in statistics education (GAISE) report: A curriculum framework for PreK-12 statistics education*. The American Statistical Association (ASA). Retrieved from http://www.amstat.org/education/gaise/

Garofalo, J. (1993). Mathematical problem preferences of meaning-oriented and number-oriented problem solvers. *Journal for the Education of the Gifted, 17,* 26-40.

Gürbüz, R. (2008). Olasılık konusunun öğretiminde kullanılabilecek bilgisayar destekli bir material [A computer aided material for teaching 'Probability' topic]. *Mehmet Akif Ersoy University Journal of Education Faculty, 8*(15)*,* 41-52.

Henson, K. T. (1993). *Methods and strategies for teaching in secondary and middle schools*. London, UK: Longman.

Hirsch, L. S., & O'Donnell, A. M. (2001). Representatives in statistical reasoning: Identifying and assessing misconceptions. *Journal of Research Mathematics Education, 28*(1)*,* 96-105.

Hong, E. (1993). *Mental models in word problem solving: An analysis of Korean elementary students***.** Paper presented at the Annual Meeting of the American Educational Research Association, Atlanta.

Johnson, B., & Christensen, L. (2004). *Educational research: Quantitative, qualitative, and mixed approaches* (2nd ed.). Boston, MA: Pearson.

Kafoussi, S. (2004). Can kindergarten children be successfully involved in probabilistic tasks. *Statistics Education Research Journal*, *3*(1), 29-39.

Keith, D. (2000). Finding your inner mathematician. *Chronicle Of Higher Education, 47*(5), 5-6.

Kirk, S. A., & Gallagher, J. J. (1989). *Educating exceptional children*. Boston, MA: Houghton Mifflin.

Knepper, W., Obrzut, E. J., & Copeland, P. E. (1993). Emotional and social problem-solving thinking in gifted and average elementary school children. *The Journal of Genetic Psychology, 142*(1), 25-30.

Lester, F. K., Jr., Garofalo, J., & Kroll, D. L. (1989). *Self confidence, interest, beliefs and metacognition: Key influences on problem-solving behavior.* In D. B. Mcleoad & V. M. Adams (Eds.), *Affect and mathematical problem solving: A new perspective* (pp. 75 – 88). New York, NY: Springer-Verlag.

Mason, J., Burton, L., & Stacey, K. (1998). *Thinking mathematically*. London: Addison-Wesley.

Mayer, R. E. (1982). The psychology of mathematical problem solving. In F. K. Lester & Garofalo (Ed.), *Mathematical problem solving: Issues in research* (pp. 1-13). Philadelphia, PA: Franklin Institute Press.

Meyen, E. L., & Skrtic, T. M. (1988). *Exceptional children and youth.* Denver, CO: Love.

Mills, J. (2002). Using computer simulation methods to teach statistics: A review of the literature. *Journal of Statistics Education, 10*(1). Retrieved from www.amstat.org/publications/jse/v10n1/mills.html

Montague, M., Bos, C. S., & Doucette, M. (1991). Affective, cognitive, and metacognitive attributes of eighth-grade mathematical problem solvers. *Learning Disabilities Research and Practice*, *6*(3), 145-151.

Munisamy, S., & Doraisamy, L. (1998). Levels of understanding of probability concepts among secondary school pupils. *International Journal of Mathematical Education in Science and Technology, 29*(1).

Naglieri, J. A. & Das, J. P. (2005). Planning, Attention, Simultaneous, Successive (PASS) Theory: A revision of the concept of intelligence. In D. P. Flanagan and P. L. Harrison (Eds.), *Contemporary intellectual assessment* (pp. 120-135). New York, NY: The Guilford Press.

National Council of Teachers of Mathematics. (2000). *Principles and standards for school mathematics*. Reston/VA: Author.

O' Connell, A. A. (1999). Understanding the nature of errors in probability problem-solving. *Educational Research and Evaluation*, 5 (1), 1-21.

Pajares, F. (1996). Self-efficacy beliefs and mathematical problem-solving of gifted students, *Contemporary Educational Psychology, 21*(25), 325–344.

Polaki, M. V. (2002). Using instruction to indentify key features of basotho elementary students' growth in probabilistic thinking. *Mathematical Thinking and Learning, 4*(4), 285-313.

Punch, K. F. (2013). *Introduction to social research: Quantitative and qualitative approaches* (3th ed.). London, UK: Sage.

Putter-Smits, L. G. A., Taconis, R., & Jochems, W. M. G. (2013). Mapping context-based learning environments: The construction of an instrument. *Learning Environments Research*, *16*(3), 437-462.

Rose, D. E. (2012). Context-based learning. In N. Seel (Ed.), *Encyclopedia of the sciences of learning* (pp. 799-802). New York, NY: Springer.

Santos-Trigo, M. & Cristóbal-Escalante, C. (2008). Emerging high school students´ problem solving trajectories based on the use of dynamic software. *Journal of Computers in Mathematics and Science Teaching, 27*(3), 325-340.

Shore, B. M. & Dover, A.C. (1987). Metacognition, intelligence, and giftedness, *Gifted Child Quarterly, 31*(1), 37-39.

Sriraman, B. (2003). Mathematical giftedness, problem solving, and the ability to formulate generalizations. *The Journal of Secondary Gifted Education, 14,* 151-165.

Sisk, D. (1987). *Creative teaching of the gifted.* New York, NY: McGraw-Hill.

Strauss, A., & Corbin, J. (1990). *Basics of qualitative research*. Newbury Park, CA: Sage.

Suzuki, K. (1998). *Measuring "to think mathematically": Cognitive characterization of achievement levels in performance-based assessment.* (Unpublished doctoral dissertation, University of Illinois at Urbana-Champaign).

Taconis, R., Ferguson-Hessler, M. G. M., & Broekkamp, H. (2001). Teaching science problem solving: An overview of experimental work. *Journal of Research in Science Teaching, 38*(4), 442-468.

Tasdemir, A. (2008). Matematiksel Düşünme Becerilerinin İlköğretim Öğrencilerinin Fen Ve Teknoloji Dersindeki Akademik Başarıları, Problem Çözme Becerileri Ve Tutumları Üzerine Etkileri [The influences of mathematical thinking skills on the success, problem solving skills and attitudes of students in primary schools at science and technology course] (published doctoral thesis). Gazi University, Ankara, Turkey.

Tatar, E. (2012). The effect of dynamic mathematics software on achievement in mathematics: The case of trigonometry. *Energy Education Science and Technology Part B*. *4*(1), 459-468.

Tatsis, K., Kafoussi, S., & Skoumpourdi, C. (2008). Kindergarten children discussing the fairness of probabilistic games: The creation of a primary discursive community. *Early Childhood Education Journal, 36,* 221-226.

Tuna, A. (2011). Trigonometri Öğretiminde 5E Öğrenme Döngüsü Modelinin Öğrencilerin Matematiksel Düşünme Ve Akademik Başarılarına Etkisi [The effect of the 5E learning cycle modelon students' mathematical thinking and academic achievements in teaching trigonometry]. (Published doctoral thesis), Gazi University, Ankara, Turkey.

Umay, A. (2007). *Eski arkadaşımız okul matematiğinin yeni yüzü [Our old friend, the new face of school mathematics]***.** Ankara, Turkey: Aydan Web Facilities.

Yesildere, S., & Turnuklu, E. (2007). Öğrencilerin matematiksel düşünme ve akıl yürütme süreçlerinin incelenmesi *[Examination of Students' Mathematical Thinking and Reasoning Processes]. Ankara University Journal of Faculty of Educational scıence, 40*(1), 181–213.

Yildiz, A. (2013). Ders imecesinin matematik öğretmenlerinin problem çözme ortamlarında öğrencilerinin üstbilişlerini harekete geçirmeye yönelik davranışlarına etkisi [The effect of lesson study to the behaviours of mathematics teachers intended to promote the metacognitions of the students in the problem solving environment], (Unpublished doctoral thesis), Karadeniz Technic University. Trabzon. Turkey.

Yildiz, A., & Baltaci, S. (2015). İlköğretim matematik öğretmen adaylarının problem kurma etkinlikleri ile olasılığa yönelik bilgilerinin incelenmesi *[Researching primary preservice mathematics teachers' knowledge of probability with problem posing activities]. Ahi Evran University Journal of Kırsehir Education Faculty (KEFAD), 16* (1), 201-213.

Yildiz, A., Baltaci, S., & Guven, B. (2011). Metacognitive behaviours of the eighth grade gifted students in problem solving process. *The New Educational Review, 26*(4)*,* 248-260.

Watson, J. M. (2006). *Statistical literacy at school, growth and goal*. London, UK: Erlbaum.

Wieczerkowski, W., Cropley, A. J. & Prado, T. M. (2000). *Nurturing talents/gifts in mathematics*. In K. A. Heller, F. J. Monks, R. J. Sternberg, & R. F. Subotnik (Eds.), *International handbook of giftedness and talent education* (pp. 413-425). Oxford, UK: Pergamon.

APPENDIX

Problems Used in the Research

1. **Problem:** Ali's father forms a dart board by drawing a rectangle whose perimeter is 24 units and a circle tangent to the two sides of this rectangle. Ali's father gives darts arrows to Ali wants him to hit the thrown arrow to the dart board. Meanwhile, assuming that Ali hits the dart board, Ali's father tries to calculate the probability of arrow's hitting outside the circle with his mathematical knowledge.

Please guide Ali's father through using the following quadrilaterals?

a. For square;

b. For rectangle;

c. For parallelogram

2. **Problem:** Ali and Veli play that game when they have free time. In this game, both of them try to hit the target they determined from the same distance. Ali hits the target 40 times out of an average 100 shots while Veli hits 75 times out of an average 100 shots.

When Ali and Veli hit the target;

Calculate,

a. the probability of being hit by both

b. the probability of being hit by one?

3. **Problem:** Mehmet who named the bags as A and B puts 3 white and 4 black balls in bag A while 2 white 3 black balls in bag B. Then, he wants Akif to draw a ball from bag A, put it in bag B and draw a ball form bag B. What is the probability that the white ball drawn from bag B by Akif?