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COMMUTATORS OF MARCINKIEWICZ INTEGRALS ASSOCIATED WITH SCHRÖDINGER OPERATOR ON GENERALIZED WEIGHTED MORREY SPACES

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Abstract. Let $L=-\Delta+V$ be a Schrödinger operator, where Δ is the Laplacian on $\mathbb{R}^n$, while nonnegative potential V belongs to the reverse Hölder class. Let also $\Omega\in L_q(S^{n-1})$ be a homogeneous function of degree zero with $q>1$ and have a mean value zero on S^{n-1}. In this paper, we study the boundedness of the Marcinkiewicz operators $\mu^L_{j,\Omega}$ and their commutators $\mu^L_{j,\Omega,b}$ with rough kernels associated with Schrödinger operator on generalized weighted Morrey spaces $M_{p,\varphi}(w)$. We find the sufficient conditions on the pair (φ_1,φ_2) with $q'<p<\infty$ and $w\in A_{p/q'}$ or $1<p<q$ and $w^{1-p'}\in A_{p'/q'}$ which ensures the boundedness of the operators $\mu^L_{j,\Omega}$ from one generalized weighted Morrey space $M_{p,\varphi_1}(w)$ to another $M_{p,\varphi_2}(w)$ for $1<p<\infty$. We find the sufficient conditions on the pair (φ_1,φ_2) with $b\in BMO(\mathbb{R}^n)$ and $q'<p<\infty$, $w\in A_{p/q'}$ or $1<p<q$, $w^{1-p'}\in A_{p'/q'}$ which ensures the boundedness of the operators $\mu^L_{j,\Omega,b}$, $j=1,\dots,n$ from $M_{p,\varphi_1}(w)$ to $M_{p,\varphi_2}(w)$ for $1<p<\infty$. In all cases the conditions for the boundedness of the operators $\mu^L_{j,\Omega}$, $\mu^L_{j,\Omega,b}$, $j=1,\dots,n$ are given in terms of Zygmund-type integral inequalities on (φ_1,φ_2) and w, which do not assume any assumption on monotonicity of $\varphi_1(x,r)$, $\varphi_2(x,r)$ in r.

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Paper JMI-10-77

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